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DEVELOPMENT OF DEMAND FORECASTING MODELS
FOR THE UNIT AIR CONDITIONER
INDUSTRY IN THE U.S.A.

A THESIS

Presented to

the Faculty of the Graduate Division

by

A. S. Rama Sastry

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Approved:

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Date approved by Chairman: May 30, 1963

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SUMMARY

The objective of this study is to develop suitable mathematical models to predict the demand for room air conditioner units.

Linear regression models were formed by formulating manufacturers' shipments as a function of disposable personal income, change in income from previous year, and number of dwelling units. Even though all of the three models developed showed a high multiple correlation of 0.92 and more, two models were selected for reasons of plausibility. The selected models gave demand for room air conditioner units as a linear function of disposable personal income and change of income from previous year, both expressed in 1954 prices, and number of wired dwelling units.

Although the selected models have a high multiple correlation, explaining much of the demand, they also have a large standard error. The models, therefore, are not as reliable as desired. Their error may be due to the fact that the industry is growing, and also may be due to the changes in weather conditions not directly included as a separate factor in the models.

The models also do not reflect the swings in the yearly shipments of air conditioner units. The predicted

values are increasing with time and the models do not predict the turning points in the sales of units.

The main argument against the use of the regression models is that they are not developed on the basis of a structured economic theory. They do not distinguish the characteristic market structure for consumer durable goods, viz., replacement demand and new-owner demand. For the above reasons, linear regression models were abandoned in favor of a model based on a well-structured demand theory.

A theory of demand for electrical appliances was developed on the basis of the maximum-ownership-level concept formulated by Roos and Von Szeleski in their classic study, The Dynamics of Automobile Demand. The market is divided into two segments:

1. Replacement market.
2. New-Owner market.

Replacement demand was calculated by using theoretical survival functions. New-owner demand is determined as a function of purchasing power and number of wired homes. Purchasing power is calculated from disposable personal income, credit extended (excluding automobiles), and price index for house furnishings expressed at the 1947-1949 level. Weather conditions are not included in the model.

The model gave a good prediction for the demand for room air conditioner units. The assumption that maximum-ownership level can be considered as a constant for the period under study is made while determining the parameter "a". The assumption was tested and proved to be acceptable. It was also proved that small changes in the value of parameter "a" do not affect the model. Therefore, small errors in the value of "a" are permissible.

To test how good the model is, sales of air conditioner units are predicted for the years 1960 and 1961, and the results are encouraging. The model developed is for the demand at industry level and, therefore, is likely to have large variations. The model gave predictions accurate enough to be acceptable at industry level.

CHAPTER I

INTRODUCTION

In industry, forecasts are made for a variety of purposes, such as cash budgeting, capital expenditures, production and inventory planning, establishing sales quotas, and the like. The objective of this study is to develop suitable mathematical models for predicting future demand for room air conditioner units in the United States. A demand theory will be developed for electrical appliances and this theory will be applied to develop a demand model for the above industry.

Methods of Forecasting

There are three kinds of evidence upon which to base forecasts for consumer durable goods:

a. Relationships between different sets of historical data.

b. Current opinions.

c. Statements of current intentions on the part of the agents whose behavior is the subject of the forecast.

Some forecasters lean heavily on one or another approach while many others rely on more than one of these kinds of evidence, and in addition, use their intuitive judgments.

An outstandingly good example of a forecast is the article "The Changing Automobile Market" published in Fortune in September, 1953. This article analyses various techniques of forecasting and various specific forecasts, and finally makes a plausible forecast of its own.

Historical Data

There have been attempts to utilize historical data to develop regression models for the purpose of forecasting. The typical procedure is to fit least-square expressions to time series data with sales as the dependent variable and independent variables represented by various economic aggregates, usually including some measure of income and some indication of population or particular relevant changes in demographic data. A widely referred example of such regression analysis is that of Roos and Von Szeliski[2] on the demand for automobiles. In this study the demand for automobiles is developed as a function of (1) supernumerary income (which is the money available to consumers over and above their necessary expenditures, like food and clothing), (2) an index of automobile prices, (3) the stock of cars in use, (4) maximum ownership level (which is a function of supernumerary income, durability and the number of families), and (5) replacement pressure (which is a function of income, price and the theoretical scrapping rate).

Another interesting example of regression analysis is the work done by Miller[3], and Spencer and Mattheis[4]. Miller's presentation combines cross-section information and time series data to make use of successive cross sections. He estimates the influence of income, other non-price variables and, less satisfactorily, price on aggregate demand for refrigerators.

Spencer and Mattheis[4], in their article, develop the framework for an econometric model connecting sales, price, credit conditions, income, and use facilities following the same basic theory or maximum ownership concept of Roos-Von Szeliski. This framework is applicable to any durable consumer goods and they demonstrate their accuracy by constructing an econometric model for the demand for washing machines.

The Department of Commerce published two studies by Jacobs and Winston[5], and Atkinson[6,7] in Survey of Current Business dealing with demand for models for furniture and other consumer durable goods. They achieved high correlations of 0.95 and more for the relationships between sales of a variety of consumer durable goods and independent variables such as income, income change, number of households, and prices. The main limitation of these studies by regression analysis is the fact that their usefulness is limited by the accuracy of the estimates of independent variables. The problem is especially acute when elasticity of demand with respect to that variable is greater than unity.

Current Opinions

The Wall Street Journal publishes reports of interviews with scattered dealers about the changes in demand for goods. From these reports the movement of demand can be evaluated. However, the evaluation is difficult and not precise since the sampling and interviewing techniques are not reported so that their standards can be examined.

Consumer Intentions

There is a growing conviction among businessmen that consumers plan their major purchases substantially in advance and that the information on these plans can be secured with accuracy by means of personal interviews. The Federal Reserve Bulletin publishes this information with interpretations, and also Survey of Consumer Finances contains the data. Survey Research Center of Michigan University conducts these intentions surveys and publishes them.[8] The use of intentions data for forecasting is not well developed and the data are largely limited to forecasts for one year. However, these data have been useful to predict the change in direction in the volume of consumer durable goods expenditures.

In the next chapters of this study a framework will be established for the development of demand models for room air conditioner units in the USA. Static regression and dynamic models to predict demand will be developed. These models will connect yearly sales and various other factors, such as income, credit, prices, and relevant demographic factors. The dynamic model will be tested for accuracy, and sensitivity analysis will be conducted on an analog computer to see the effect of changing the proportionality coefficient in the model predicting new owner demand.

CHAPTER II

REGRESSION MODELS FOR THE DEMAND FOR ROOM AIR CONDITIONER UNITS

The objective of this analysis is to develop several multiple regression models to predict the demand for room air conditioner units in the USA. The manufacturers' shipments of units were obtained from the Electrical Merchandising and they constitute the dependent variable. The independent variables used are personal disposable income, changes in personal disposable income from previous years, and number of electrically wired dwelling units. The former two variables represent the economic conditions while the latter represents the growth variable.

Basic Demand Factors

The most important factor influencing the demand for any electrical appliance is the level of economic conditions which, in turn, represents the purchasing power. Disposable personal income series obtainable from the Survey of Current Business was selected to represent the economic conditions. To make the data comparable, it is expressed in 1954 prices.

In addition to the effect of income levels, demand for electrical appliances is also affected by the direction in which the income has been changing from the previous year. In other words, for any given current income purchases will be greater when the income is rising than when it is falling. This phenomenon may be due to more confidence regarding future income prospects when income has been rising in the immediate past, and a corresponding pessimism associated with falling income.

There is usually some lag in spending by the consumers for many major items like appliances. Consequently, when income is rising, cash is a little freer and consumers may tend to trade in their old appliances and buy new models sooner than in a period of stable income. When income is falling, consumers may feel pinched for cash and tend to defer purchasing.

The changes in population growth and family formation are the other demand factors which influence the demand for appliances significantly. They form or represent the growth factors. The electrical appliances require wired homes for their consumption. The number of wired homes also reflects population growth and family formation. Therefore, it can be used as a proxy variable for these two factors and forms another independent variable representing demographic and use-facility factor. The data for the number of electrically wired homes is obtained from <u>Electrical Merchandising</u>. All the relevant data is shown in Table 1. The prewar data is neglected since it is not considered to

Table 1. Comparison of Regression Models

t	Sales in Thousands of Units	I_t in Billions	ΔI_t in Billions	H_t in Millions	T	S_t From Model I	S_t Model II
1946	30	209.9	- 1.9	29.4	1	92	- 67
1947	43	201.1	- 8.8	31.2	2	?	- 62
1948	74	211.5	10.4	33.1	3	126	88
1949	89	213.8	2.3	35.2	4	158	207
1950	201	231.0	17.2	37.2	5	423	425
1951	238	237.0	6.0	39.1	6	535	580
1952	380	243.6	6.6	40.9	7	642	676
1953	1,045	255.0	11.4	42.4	8	830	879
1954	1,353	256.9	1.9	43.6	9	854	954
1955	1,275	273.4	16.5	44.8	10	1,130	1,038
1956	1,828	286.9	13.5	46.1	11	1,346	1,320
1957	1,586	293.8	6.9	48.6	12	1,453	1,498
1958	1,673	296.3	2.5	49.4	13	1,491	1,561
1959	1,660	310.6	14.3	50.6	14	1,730	1,726
1960	1,580	319.0	8.4	51.7	15	1,860	1,856
1961	1,500	321.6	2.6	55.4	16	1,899	2,044

be reliable.

Demand for air conditioner units also depends on weather conditions. Hot, humid weather can be considered as a use facility. But there is no reliable way of quantifying hot and humid conditions. Besides, the weather conditions are random in nature. They have no predictable trend. The weather conditions, therefore, are not included as a specific factor in the model, but they form a part of the unexplained variation. Also, there is conflicting evidence with regard to the effect of weather conditions on demand for air conditioners. In the surveys for demand for air conditioner units conducted by DuPont and Hot Point[10], DuPont survey indicated 67 per cent of owners attributing the purchase to weather conditions while Hot Point indicated only 12.7 per cent of owners attributing weather as a reason for purchase. Moreover, with increasing standards of living, an air conditioning unit is becoming more and more a necessary appliance, like a range and refrigerator, and hence ownership depends more on economic conditions rather than on weather conditions. For all the above reasons, it is not explicitly included in the study but it shows up in unexplained variation.

Multiple Regression Models

The proposed multiple regression models are developed with the assumption that manufacturers' sales of air conditioner units are a linear function of the independent variables, viz.,

$$S_t = b_0 + b_1 I_t + b_2 \Delta I_t + b_3 H_t + \epsilon_t$$

where

S_t = manufacturer's shipments in thousands of units

I_t = disposable personal income at 1954 price level in billions of dollars

ΔI_t = change in disposable personal income in billions of dollars from period t-1 to t

H_t = number of wired dwelling units in millions.

T = time in years with $T = 1$ at $t = 1946$.

t = time (e.g., $t = 1954$); $1946 \leq t \leq 1959$.

ϵ_t = random error independent of I_t, ΔI_t and H_t.

b_i = coefficients.

Models were developed as shown in Table 2.

Table 2. Regression Models

No.	Model	Multiple Correlation R	Standard Error
I	$S_t = -3296.1+16.15I_t+0.66\,\Delta I_t$	0.9274	288.5
II	$S_t = -3227.1+9.05I_t-1.399\,\Delta I_t + 42.77H_t$	0.9315	291.8
III	$S_t = -5030.8-7.11I_t+9.06\,\Delta I_t$ $19.9916 \text{ Log } T$	0.9351	302.3

All the above models have quite high multiple correlation coefficients. But model III does not seem to be plausible. In model III, sales seem to decrease with higher income levels. This is quite improbable. Errors may be due to high correlation between disposable personal income and number of wired households, as both of them have a secular growth trend. The standard error value in model III is higher than in model I.

For the above reasons, model III is discarded and models I and II are chosen to predict demand for room air conditioners. In these models, the standard error is too high to be reliable. The reason may be due to the fact that the air conditioner industry is a growth industry and is becoming mature. The large spurts in sales in 1953 and 1956 and the fall in sales in years of higher income levels, may be the reason for the model not showing good results. This will be the case for most industries in their growth stages. Their growth is not attributable to any particular factors. As they grow, they become mature and respond well to economic and other factors. Moreover, the violent fluctuations in sales of later years may be due to weather conditions which are included in the error term and hence the larger value of standard error. The sales calculated from the two models I and II are compared with actual sales in Figure 1.

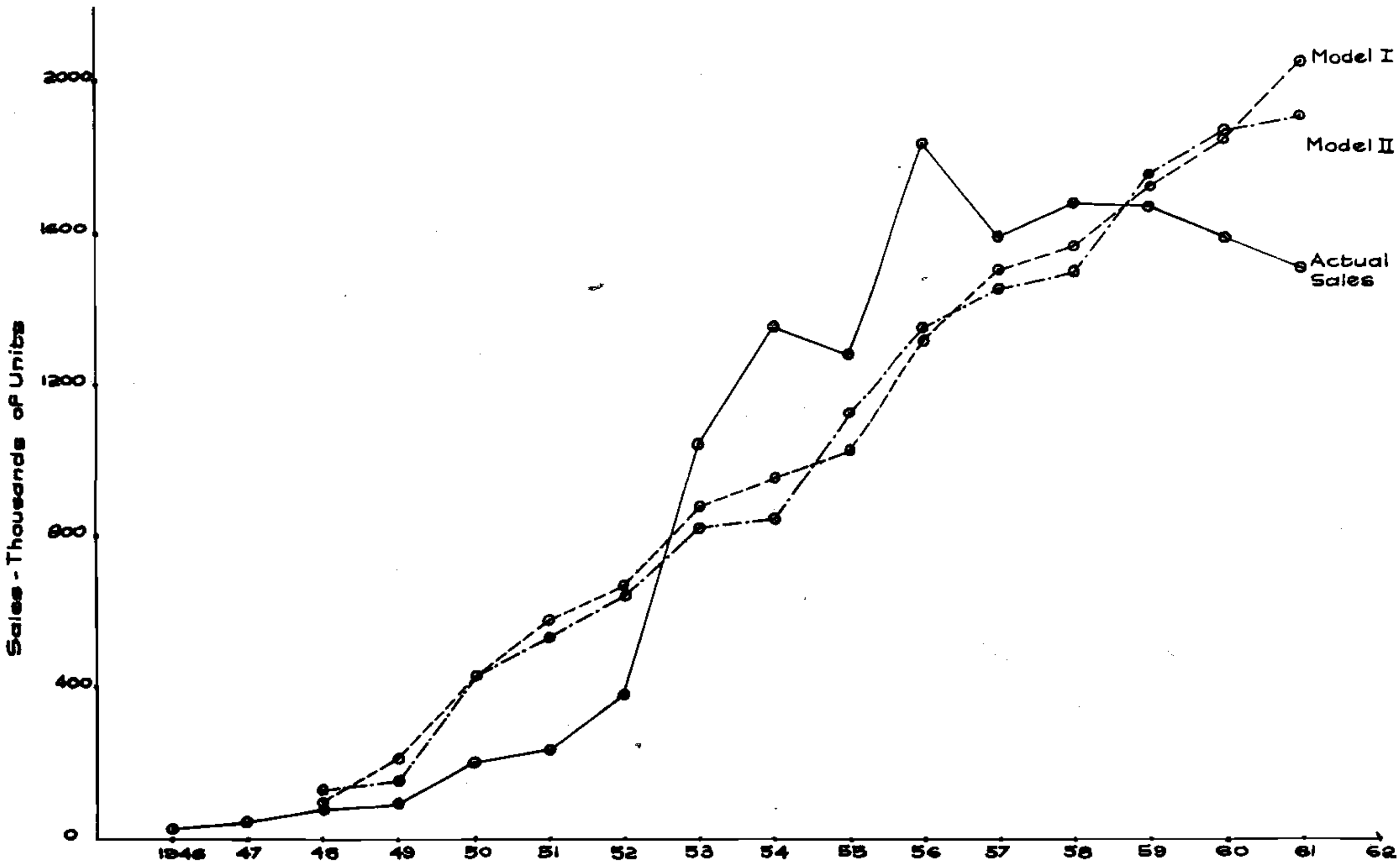


Figure 1. Comparison of Regression Models

Limitations of Regression Models

The regression models that have been developed in forecasting literature are similar to those developed above. They are not based on sound economic theory, in addition to other limitations mentioned in Chapter I. Often different variables and different combinations of variables form models that can produce an equally satisfactory fit, and, therefore, statistical goodness of fit should not form a sufficient criterion of adequacy of a model. Moreover, the statistical tests of goodness of fit will not distinguish between those models made up of purely empirical co-variations and those which are casually significant formulations. Therefore, what is needed for the best formulation of forecasting models is the development of an economic theory which will justify the relationship found. An attempt will be made in the next chapter to develop a well-structured economic theory for the demand for electrical appliances which can be used to develop an econometric model for predicting demand for room air conditioner units in the USA.

CHAPTER III

A DEMAND THEORY FOR ELECTRICAL APPLIANCES

In the last chapter the deficiencies of linear multiple regression models were mentioned. In this chapter the theoretical framework to forecast production demand will be developed on the basis of maximum ownership concept developed by Roos and Von Szeliski.

Essentially, for any durable consumer product, the framework for demand theory consists of a set of purchase characteristics defining consumption environment, and a growth model or basic equation to express the relationship between new demand, replacement demand, and total demand. The electrical appliances, being durable, pose a problem of estimating the replacement needs (or demand). Before going further on this subject, purchase characteristics will be defined.

Purchase Characteristics

The purchase characteristics are defined as those features that condition the consumption of the appliance and hence the willingness of people to buy that particular appliance. There are three purchase characteristics for electrical appliances:

1. Time-use characteristics.

2. Use-facilities characteristics.

3. Demographic characteristics.

The significance of each of these characteristics as demand determinants is explained below.

Time-Use Characteristics

Electrical appliances are consumer durables. They are not consumed in a single act or day, but extend their service over a period of time. This raises a particular problem of forecasting, since it is not known whether the consumers will continue to use the old appliance by repairing it, if necessary, or will replace it with a new one. The latter alternative involves the decision between:

1. scrapping the old appliance,

2. trading it in, or

3. selling it as used one.

Whatever the decision, it depends as much on noneconomic factors, such as social status or desire for prestige, as on economic factors which include product obsolescence and income.

Use-Facilities Characteristics

The electrical appliances require special facilities for their consumption. Wired homes are necessary to install and operate them. As an example, wired homes and leisure time are required for the consumption of television sets. Air conditioners require a wired home plus hot or humid weather conditions. Wired homes, therefore, form a use-facility for the consumption of electrical appliances. Leisure time and weather conditions also form use-facilities for television and air conditioners respectively.

Demographic Characteristics

The electrical appliances are consumed by more than one person. For example, a family consumes a television set, a washing machine or an air conditioner unit. Therefore, the decision to buy an appliance depends on the demographic factors, such as family size, age group, and rate of family formation. As explained in the previous chapter, the number of wired homes can be used to represent both a use-facility characteristic and a proxy variable for the demographic characteristics mentioned above. In this way one less independent variable is employed while the effect is that of maintaining, if not improving, the statistical significance of results as well as forecasting accuracy.

Market Structure

The above characteristics give rise to a particular type of market structure with the demand segments being replacement demand and new-owner demand. The replacement demand tends to form quite a significant portion of the market, constituting half of or over half of total demand for long established products like refrigerators. Moreover, replacement demand tends to grow with consumer stocks. Also, purchasing power has the effect of increasing scrapping rates, and thus increase replacement demand as it increases. In general, the scrapping rates depend mainly on purchasing power and production, and to a lesser extent on physical construction and operating costs.

Basic Equation

In the light of above discussion, the basic demand equation can be formulated as:

$$S_t = R_t + N_t$$

where S_t = sales of appliance in any period t,

R_t = replacement demand in any period t,

N_t = new-owner demand or increase in existing stock in any period t.

The two independent variables are thus N_t and R_t. In any particular study, they can be separately estimated and a forecast of total demand can be made. R_t can be estimated from replacement data, if available, or from consumer survey that can be made to obtain necessary data, or can be calculated from life expectancy tables or survival functions. N_t is estimated from growth model discussed below.

Growth Model

New-owner demand is the change in consumer stock of the goods being studied in unit time. In symbolic form

$$N_t = \frac{\Delta Y_t}{\Delta t}$$

where ΔY_t represents change in consumer stock and Δt is unit time.

N_t is estimated on the basis of certain concepts borrowed from population theories. Demographers have long studied the growth and decline of population by the use of logistic functions. If the stock of electrical appliances is considered as a population exhibiting both 'birth' and 'death' characteristics, the following axiom appears plausible:

> At any given time, t, there exists a "maximum" or "optimum ownership level" towards which consumers are constantly adjusting their stock of durable goods, yet depending on the existing economic and cultural conditions.[9]

It is maximum in the sense that it is a level toward which the actual volume of consumption of a particular good is gravitating.

Statistically, the maximum ownership, $M(t)$, is a function of purchasing power (PP), demographic factors (D), and replacement factor (R). Hence, it is not a constant but a dynamic function of time as PP, D, and R are changing with time. Symbolically,

$$M(t) = f(PP_t, D_t, R_t)$$

The rate of growth, $\Delta Y_t/\Delta t = N_t$ is proportional to the existing stock of the appliance Y_t and to the potential expansion of stock, $\left[M(t) - Y_t\right]$. Therefore, assuming a multiplicative relationship,

$$N_t = \frac{\Delta Y_t}{\Delta t} = a\left(M(t) - Y_t\right) Y_t = f(PP_t, D_t, R_t, Y_t)$$

where "a" is the parameter.

The full development of this model to forecast demand for room air conditioner units will be shown in the next chapter and its success as a predictive model will be analysed.

CHAPTER IV

DEVELOPMENT OF THE DYNAMIC MODEL

In the last chapter theoretical equations were formulated which express the demand for electrical appliances. These equations provide a guide to what parameters need to be measured when analysing the demand for the specific appliance. In other words when the form of equation is known, the problem is to find the necessary coefficients.

The basic equation for the demand prediction for room air conditioner units developed are:

$$S_t = Nt + R_t \quad \ldots\ldots\ldots\ldots \quad (1)$$

$$N_t = \frac{\Delta Y_t}{\Delta t} = aY_t \left[M(t) - Y_t \right] \quad \ldots\ldots \quad (2)$$

Where S_t = total sales of product in thousands of units.

N_t = new-owner demand, or increase in total stock in use in unit time in thousands of units.

R_t = replacement demand as measured by scrappage of old units in thousands of units.

Y_t = number in use at the end of each period in thousands of units.

t = time in years.

M(t) = maximum ownership level as a function of t in thousands of units.

These relationships provide a general framework for the analysis of demand for unit room air conditioners.

Methodology

From equation (1) and (2):

$$\frac{N_t}{Y_t} = \frac{S_t - R_t}{Y_t} = aM(t) - aY_t \quad \ldots\ldots \quad (3)$$

Therefore,

$$aM(t) = \frac{S_t - R_t}{Y_t} + aY_t = f(PP_t,\ D_t,\ R_t) \quad \ldots \quad (4)$$

where

PP_t = purchasing power.

D_t = demographic factor.

R_t = replacement factor.

Now the procedure consists in estimating the value of parameter "a," and substituting in the second term of the right hand side of equation (4), thereby obtaining values which are then correlated with $f(PP_t, D_t, R_t)$. R_t is estimated from survival tables which will be shown later, and then equation (2) is then solved for N_t. Knowing N_t and R_t, S_t is calculated.

Replacement Demand

Here the procedure adopted to estimate replacement demand will be outlined. Normally this can be done by conducting a sample survey of consumers. This is an expensive and time-consuming proposition. Hence, recourse is made to utilize theoretical survival functions. The survival coefficients are obtained (with an assumption of 14 years of maximum life or 8 years of average life) from Kimball's[10] paper in Econometrica and are shown in Table 3.

Sales Data

Manufacturers' sales figures of room air conditioner units are available from the magazine Electrical Merchandising. These figures are not the same as sales to consumers, but over a period of years manufacturers' sales would equal consumer sales. They have more cyclical fluctuations than consumer sales. Since no reliable figures of sales to consumers are available, manufacturers' sales figures are utilized in this study. Table 4 shows the manufacturers' sales in this country.

Scrappage and New-Owner Sales

The survival coefficients in Table 3 represent the proportion of a stock that will survive to a given year. For example, given a 14-year maximum life for air conditioners, the percentage of units produced in any year that will survive after 8 years if 50 per cent, and after 12 years is 1.1 per cent. These coefficients multiplied by

Table 3. Survival Coefficients

Maximum Service Life: 14 Years
Average Service Life: 8 Years

Age in Years j	α_j
1	1.000
2	1.000
3	0.998
4	0.989
5	0.957
6	0.874
7	0.716
8	0.500
9	0.284
10	0.126
11	0.043
12	0.011
13	0.002
14	0.000

Table 4. Analysis of Demand for Air Conditioner Units

Year	In Thousands of Units: Actual Sales	Replacement	New-Owner Demand	Consumers' Inventory	Percentage Change In Inventory	Inventory Adjustment	Percentage Change In Adjusted Inventory
t	S_t	R_t	$N_t=S_t-R_t$	Y_t	$\frac{N_t}{Y_t}$	aY_t	$\frac{N_t}{Y_t} + aY_t$
1946	30	0.0	30.0				
1947	43	0.0	43.0	30.0	143.3	0.114	143.4
1948	74	0.0	74.0	73.0	101.4	0.28	101.7
1949	89	0.3	88.7	147.0	60.4	0.56	61.0
1950	201	1.5	199.5	235.7	84.7	0.90	85.6
1951	238	4.6	233.4	435.2	53.7	1.67	55.4
1952	380	12.1	367.9	668.6	55.1	2.56	57.7
1953	1,045	24.5	1,020.5	1,036.5	98.7	3.98	102.7
1954	1,353	49.9	1,303.1	2,057.0	50.2	7.85	58.0
1955	1,275	68.2	1,206.8	3,360.1	35.9	12.85	48.8
1956	1,828	120.3	1,707.7	4,566.9	37.3	17.48	54.8
1957	1,586	196.3	1,389.7	6,274.6	22.2	24.00	46.2
1958	1,673	321.8	1,351.2	7,664.3	17.6	29.32	46.9
1959	1,660	513.4	1,146.6	9,015.5	12.7	34.46	46.2
1960	1,580	761.3	818.7	10,162.1	8.05	38.61	46.66
1961	1,500	1,027.5	472.5	10,980.8	4.31	41.72	46.03

Table 4. Analysis of Demand for Air Conditioner Units (Continued)

Year t	Wired Dwelling Units (In Millions) H_t	Adjusted Inventory Per Dwelling Unit $\frac{N_t}{Y_t}+aY_t \div H_t$	Deviations from Real Purchasing Power from Tbl.5	Inventory Adjusted for Real Purchasing Power from eq. (10)	Maximum-Ownership Level $a\ M(t)$
1947	31.2	4.60	0.933		
1948	33.1	3.07	0.923	1.40	46.4
1949	35.2	1.73	0.922	1.39	48.9
1950	37.2	2.30	0.968	1.23	45.8
1951	39.1	1.42	0.906	1.46	57.1
1952	40.9	1.41	0.954	1.27	51.9
1953	42.4	2.42	0.961	1.24	52.6
1954	43.6	1.33	0.941	1.30	56.7
1955	44.8	1.09	0.988	1.15	51.5
1956	46.1	1.19	1.019	1.04	48.0
1957	47.4	0.976	1.024	1.02	48.4
1958	48.6	0.965	1.014	1.06	51.5
1959	49.4	0.913	1.042	0.95	47.0
1960	50.6	0.922	1.038	0.95	48.0
1961	51.7	0.902	1.042	0.95	49.2

Table 4. Analysis of Demand for Air Conditioner Units (Continued)

Year t	$a\left[M(t) - Y_t\right]$	$N_{tc}=a\left[M(t)-Y_t\right]Y_t$	$S_{tc} = N_{tc}+R_t$	Deviation of Calculated Demand $\frac{S_{tc}}{S_t}$	From Actual Sales
1947					
1948	46.12	33.7	33.7	0.46	
1949	48.34	71.0	71.3	0.80	
1950	44.85	105.5	107.0	0.53	
1951	55.43	241.0	245.6	1.03	
1952	49.34	329.6	341.7	0.90	
1953	48.62	505.0	529.5	0.51	
1954	48.85	1,030.0	1,079.9	0.80	
1955	38.65	1,296.0	1,364.2	1.07	
1956	30.52	1,393.0	1,513.3	0.83	
1957	24.40	1,530.0	1,726.3	1.09	
1958	21.18	1,620.0	1,941.8	1.16	
1959	12.54	1,130.0	1,643.4	0.99	
1960	38.61	940.0	1,701.3	1.08	
1961	41.72	746.0	1,773.5	1.18	

sales for the appropriate year provide a means to calculate consumers' inventory. Denoting survival coefficients for jth year as α_j, we can calculate

$$Y_t = S_{t-1} \cdot \alpha_1 + S_{t-2} \cdot \alpha_2 + \cdots\cdots + S_{t-j} \cdot \alpha_j + \cdots \quad (5)$$

and

$$y_t = S_{t-1} \cdot \alpha_2 + S_{t-2} \cdot \alpha_3 + \cdots\cdots + S_{t-j+1} \cdot \alpha_j \cdots \quad (6)$$

Where

Y_t = consumer stock in hand in any period.

y_t = consumer stock that would occur in the following period if no units were sold and scrappage rates remain the same.

The actual air conditioner units survival table is constructed as shown in Appendix 1.

The next step is to calculate consumers' inventory of air conditioner units for any given year. It is the sum of sales during previous year and the inventory surviving from the previous years. These are shown in Appendix I and also in Table 4.

The scrappage volume for any year is calculated in Appendix 1 by subtracting the number of units surviving to a given year from the total inventory in previous year. Then new-owner sales will be the difference between total sales and replacement as represented by scrappage. Over a period of years new-owner sales as calculated here would equal actual sales to new owners, but in any one year the actual figure may be greater or less than calculated value. The figures analysed are actually true new-owner sales plus the short-term or cyclical difference between actual scrapping and calculated scrapping. In this way the cyclical component of the scrapping is included in the analysis.

Thus, in periods of prosperity when income is high scrapping rates would tend to become above normal, and this would show up in the study as more than normal new-owner demand. Conversely, in recession periods when income is low, scrapping rates are retarded and the result is that there is lower-than-normal new demand. Hence the new-owner figure analysed below contains all the cyclical components of total sales figure.

Determination of Parameter "a"

Previously we have developed the relationships

$$N_t = a\left[M(t) - Y_t\right] Y_t \qquad \ldots\ldots \quad (2)$$

i.e.,

$$\frac{N_t}{Y_t} = a\left[M(t) - Y_t\right] \qquad \ldots\ldots \quad (3)$$

Here it is assumed that the maximum ownership level, $M(t)$, has an upper limit within specified time periods and, hence, within such periods may be expressed as a constant, M. Now equation (3) can be rewritten as

$$\frac{N_t}{Y_t} = aM - aY_t = M_o - aY_t \qquad \ldots\ldots\ldots \quad (7)$$

Where

$$M_o = aM = \text{a constant}$$

Hence the relationship between the percentage change in units is treated as a linear function of total inventory.

In Figure 2 percentage change in consumers' inventories versus consumers' inventories is plotted. If M were actually a constant throughout the analysis period, the line would have been fitted with least squares regression. Since M(t) is not a constant but varies with the cyclic conditions, the fit should represent both the secular growth and the cyclic swings for the period as a whole. For this reason, the slope of the line fitted should be average of the slopes of the lines connecting successive years in Figure 2. The method suggested by Spencer, Clark and Heguet[11] has been adopted to fit the regression line.

Referring back to equation (7), M_o in the formula represents the point where the regression line in Figure 2 intersects the vertical axis. If M_o reverts to aM(t), then each point M_o indentifies a separate line for each

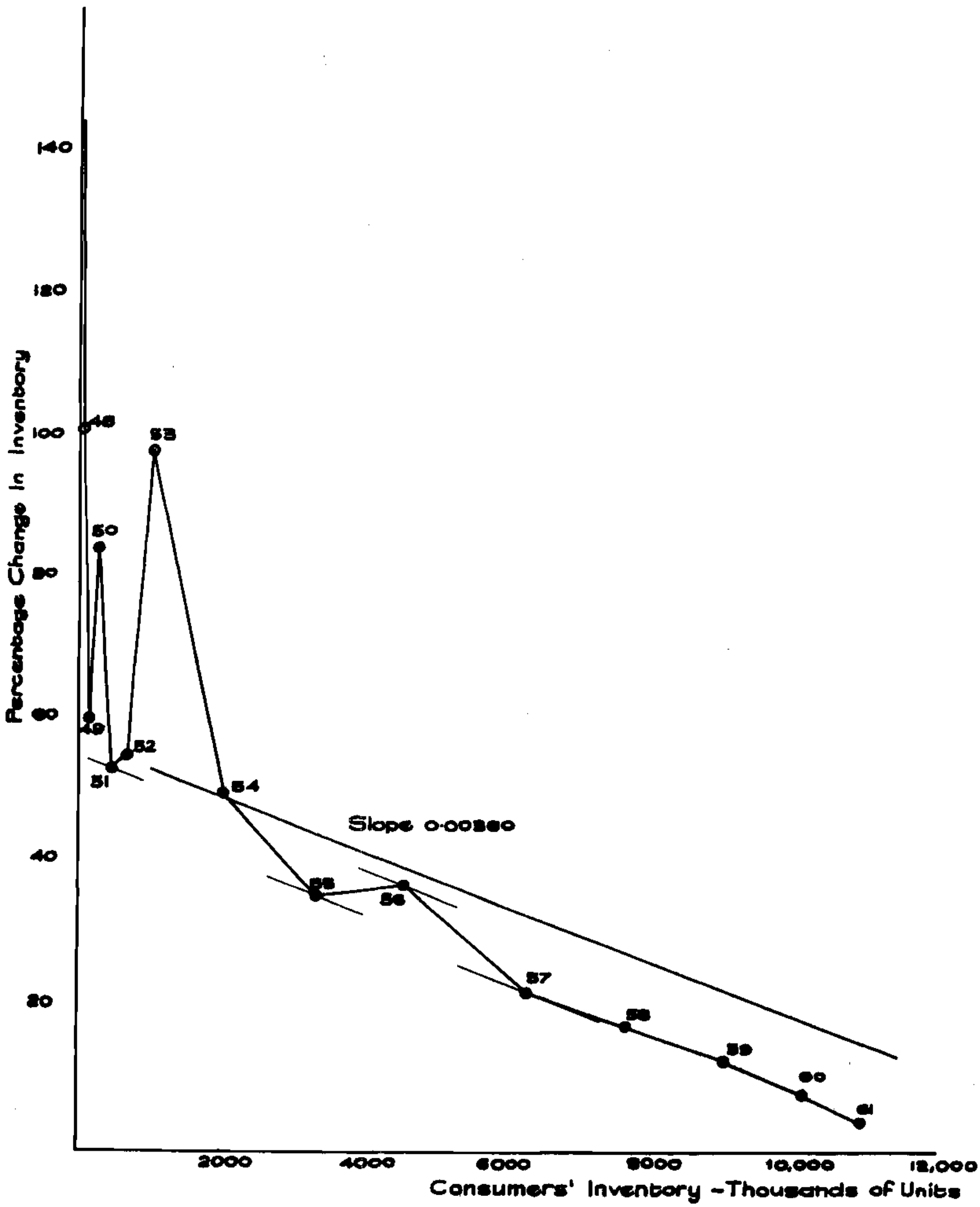


Figure 2. Percentage change in Inventory vs. Consumers' Inventory

year with the same slope as or parallel to this regression line. Any of these lines, shown in Figure 2 for each year, will intersect the vertical axis at $M_o = aM(t)$, and the horizontal axis at the maximum ownership level $M(t)$ for that period.

Determination of Maximum Ownership Level

Maximum ownership level, $M(t)$, represents the number of potential owners of a product. The factors affecting M_t are income conditions or purchasing power represented by diposable personal income, the net extension of installment credit excluding automobile credit and a price index, and wired households entering as a use-facility characteristic and as a proxy variable for demographic characteristic as explained in the last chapter. Since the factors comprising purchasing power are intercorrelated, they are combined into one synthetic variable as shown later. Their use in the analysis will help to explain any cyclical changes in rate of increase in maximum ownership level. The assumption is made that the two variables, wired households and the synthetic variable representing purchasing power, enter the relationship with $M(t)$ in a multiplicative manner. However, since the two variables are likely to be intercorrelated, it will have to be removed before the second variable is introduced.

Effect of Wired Households

Let $M_o(t)$, which is equal to $aM(t)$, represent the intersection of a regression line with X-axis in Figure 2. Rewriting equation (7), we get

$$M_o(t) = aM(t) = \frac{N_t}{Y_t} + aY_t \quad \ldots\ldots \quad (8)$$

and

$$aM(t) = f(H_t, I_t, C_t, P_t) \quad \ldots \quad (9)$$

The right side of the equation (8) represents the percentage change in consumers' inventories adjusted for inventory levels. Equation (9) shows M(t) as a function of number of households (H_t) and purchasing power represented by disposable income (I_t), credit (C_t), and price (P_t).

The problem now is to combine the two classes of variables, wired households and purchasing power. As mentioned earlier, it is assumed that they enter in a multiplicative manner. Since it is known that $M_o(t)$ is partly explained by the number of wired homes (because an air conditioner cannot be used in an unwired home), equation (8) can be rewritten as

$$\frac{\frac{N_t}{Y_t} + aY_t}{H_t} = M_1(t) = f(I_t, C_t, P_t) \quad \ldots\ldots \quad (10)$$

Where $M_1(t)$ represents purchasing power consisting of disposable income, credit, and price. Also, from Figure 3

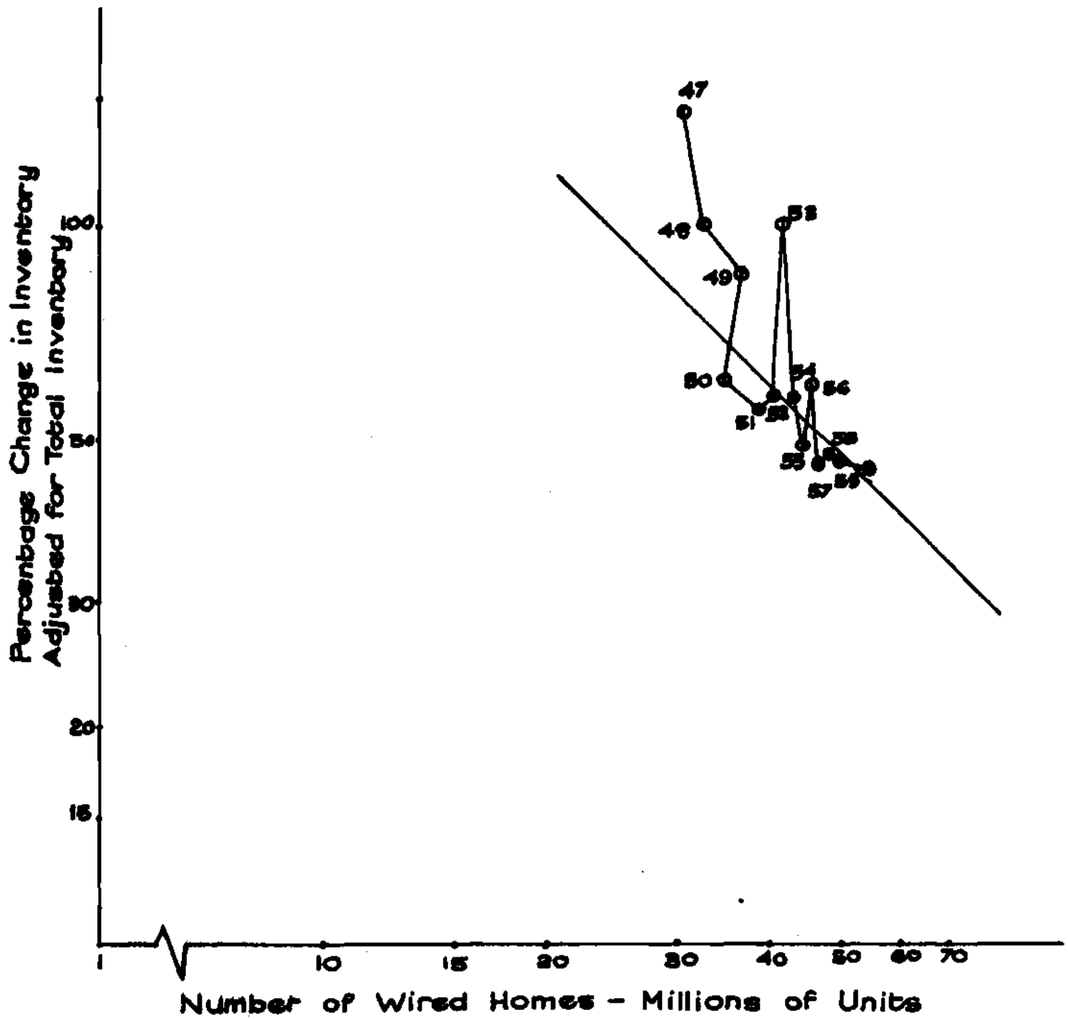


Figure 3. Percentage Change in Inventory Adjusted for Total Inventory vs. Wired Dwelling Units

we can see that both are increasing at the same rate.

Effect of Purchasing Power

A synthetic variable combining I_t, C_t, and P_t, since they are intercorrelated, will be developed here to represent the purchasing power. No data are available as regards to the cash and credit proportion of sales of air conditioners to know their proper weights while combining them. But Spencer and Mattheis[4] obtain a relationship

$$EC_t = 0.25\ I_t + 0.75\ C_t$$

where EC_t is the expenditures on consumer durables excluding automobiles.

So the weighted combination of I_t and C_t is taken as $I_t + 3C_t$. Finally

$$PP_t^1 = I_t + 3C_t$$

where PP_t^1 is purchasing power in period t.

To obtain real purchasing power, the above quantity has to be deflated by a price index. The measure chosen is the price index of house furnishings shown in Appendix 2 to get real purchasing power in 1947 - 1949 dollars. Therefore, real purchasing power

$$PP_t = \frac{I_t + 3C_t}{P_t} .$$

Since both purchasing power and wired households have a pronounced secular trend, they will be intercorrelated, and this intercorrelation has to be removed. The trend values of PP_t, i.e., PP_{tc}, are calculated by the relationship

$$\log PP_{tc} = \log \left[\frac{I_t + 3C_t}{P_t} \right]_c = M + Nt \qquad (11)$$

The coefficients M and N are found by fitting a least squares trend line. Finally,

$$PP_{tc} = f(t) = \left[\frac{I_t + 3C_t}{P_t} \right]_c = 10^{0.01818t - 33.1143} \qquad (12)$$

where t is the time expressed in years, and

c denotes calculated values.

The trend values are recorded in Appendix 2.

In order to remove this trend from real purchasing power variable PP_t, the ratio residuals $\frac{PP_t}{PP_{tc}}$ are calculated. These represent the cyclical fluctuations of real purchasing power about its trend line. They are shown in Appendix 2.

The series on adjusted inventory per dwelling unit of Table 4 are plotted against the above residuals and the resulting scatter diagram is shown in Figure 4. The regression line is drawn averaging out the slopes of the

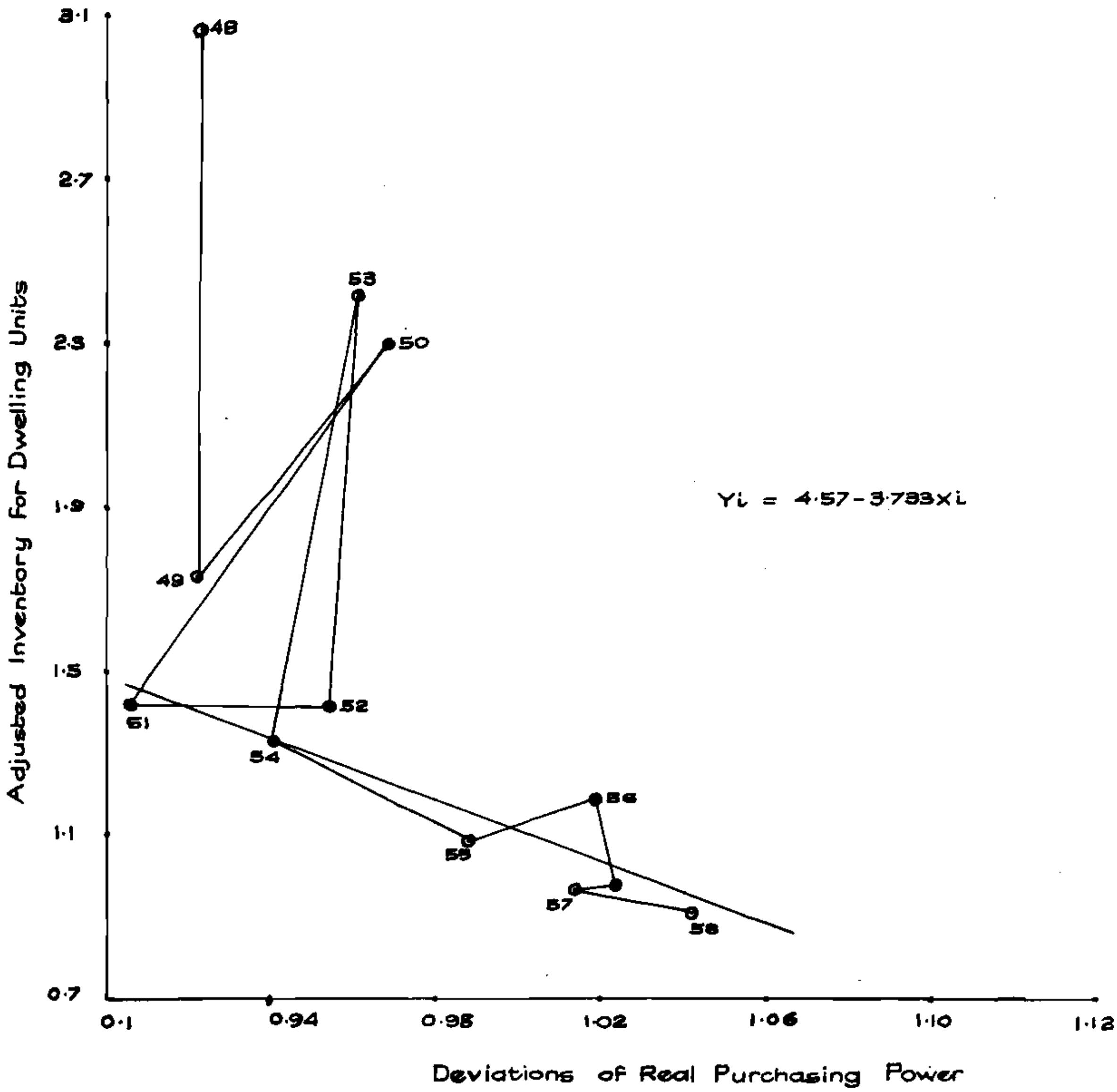


Figure 4 Adjusted Inventory for Dwelling Units vs. Deviations of Real Purchasing Power

lines connecting the points chronologically. The resulting regression line is given by

$$\bar{Y}_t = 4.57 - 3.733\ X_t$$

where $\bar{Y}_t$ = adjusted inventory per dwelling unit in thousands of units.

X_t = deviations in real purchasing power.

Finally, maximum ownership level for room air conditioner units,

$$M(t) = \frac{H_t}{a}\left[4.57 - \frac{3.733\left(\dfrac{I_t + 3C_t}{P_t}\right)}{10^{0.01818t - 33.1143}}\right] \qquad (13)$$

Therefore, new-owner demand is given by

$$N_t = a\left[M(t) - Y_t\right] Y_t$$

$$= \left\{\left[4.57 - \frac{3.733\left(\dfrac{I_t + 3C_t}{P_t}\right)}{10^{0.01818t - 33.1143}}\right] H_t - 0.0038\ Y_t\right\} Y_t \qquad (14)$$

The total demand for unit air conditioner units can be predicted by the model

$$S_{tc} = \left(Y_t - y_t\right)$$

$$+\left\{\left[4.57 - \frac{3.733\left(\frac{I_t + 3C_t}{P_t}\right)}{10^{0.01818t-33.1143}}\right]H_t - 0.0038\ Y_t\right\}Y_t \quad (15)$$

Where S_{tc} is the calculated demand,

$(Y_t - y_t)$ is scrappage rate in period t, both expressed in thousands of units.

In order to test how good the model is, the calculated value of sales were compared with actual sales by finding the ratio S_{tc}/S_t. The last column in Table 4 gives deviations of calculated demand and actual demand in a ratio form. They match well according to the standards of industry. It is not unusual in the air conditioning industry to be left with inventories forming more than 25 per cent of sales. In 1957, the inventories at factory level were 900,000 sets forming 57 per cent of sales, and in 1958, 600,000 sets forming 36 per cent of sales.

To further test the model, the demand was calculated for 1960 and 1961. Demand calculations were quite within the range of acceptable range of errors. The main reason for this error in forecast is due to the fact that the sales figures in this study were at manufacturers' level and they fluctuate violently with small changes in retail sales. Forrester, in his book Industrial Dynamics, suggests that a small percentage change in sales at retail level causes as much as 50 per cent change at manufacturers' level. Thus, it can be observed that this model has quite a good fit and forecasts demand precisely at manufacturers' level.

In the development of above model, in order to estimate the parameter "a," it was assumed that the maximum ownership level M(t) is a constant during the period (1946-61) under study. In the next chapter the assumption will be tested by conducting a sensitivity analysis.

CHAPTER V

SENSITIVITY ANALYSIS

In the last chapter a dynamic model for demand for room air conditioner units was developed and its validity was examined. The present chapter covers an analysis of the sensitivity of this model to changes in the parameters "a" and M.

From equation (2), new-owner demand is given by the relationship

$$\frac{\Delta Y_t}{\Delta t} = a\left[M(t) - Y_t\right] Y_t \qquad (2)$$

In order to estimate the parameter "a" it was assumed that M(t) can be taken as a constant for the period under study. To see the effects of changes in "a," the above model was programmed for testing on an analog computer. The programs used are shown in Figures 5 and 6.

In Figure 5 $\left[M(t) - Y_t\right]$ is treated as a function of time of the form $12.8e^{-0.25t}$ as shown in Figure 7. This choice of function was based on the values of $\left[M(t) - Y_t\right]$ calculated from Table 4, which are presented in Table 5. The computer results are shown in Figure 8. It is seen that when the parameter "a" is varied from 0.0030 to 0.0050

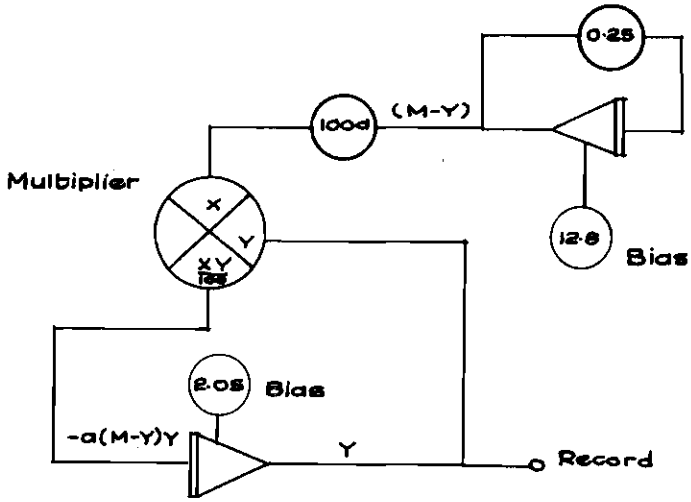


Figure 5 Sensitivity Analysis Program

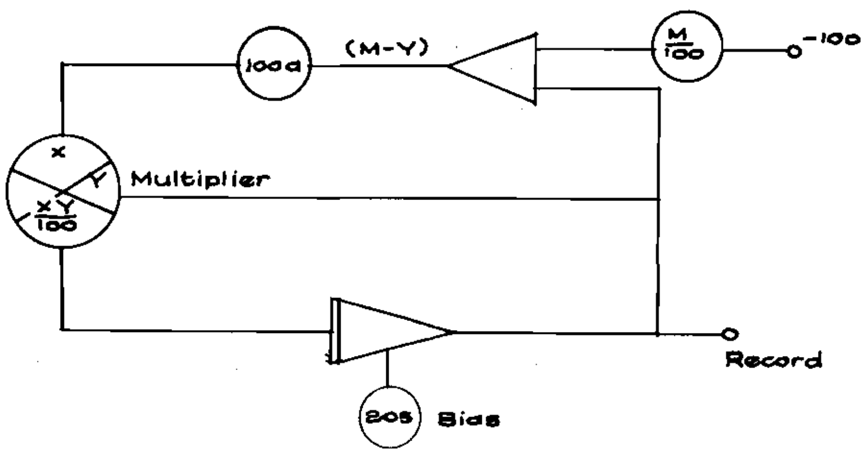
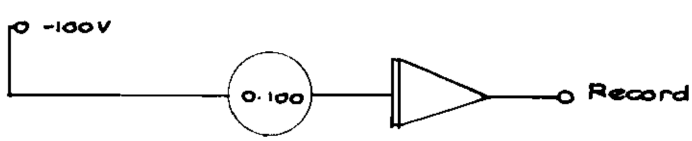


Figure 6 Sensitivity Analysis Program



Driving Function for Figures 5 & 6.

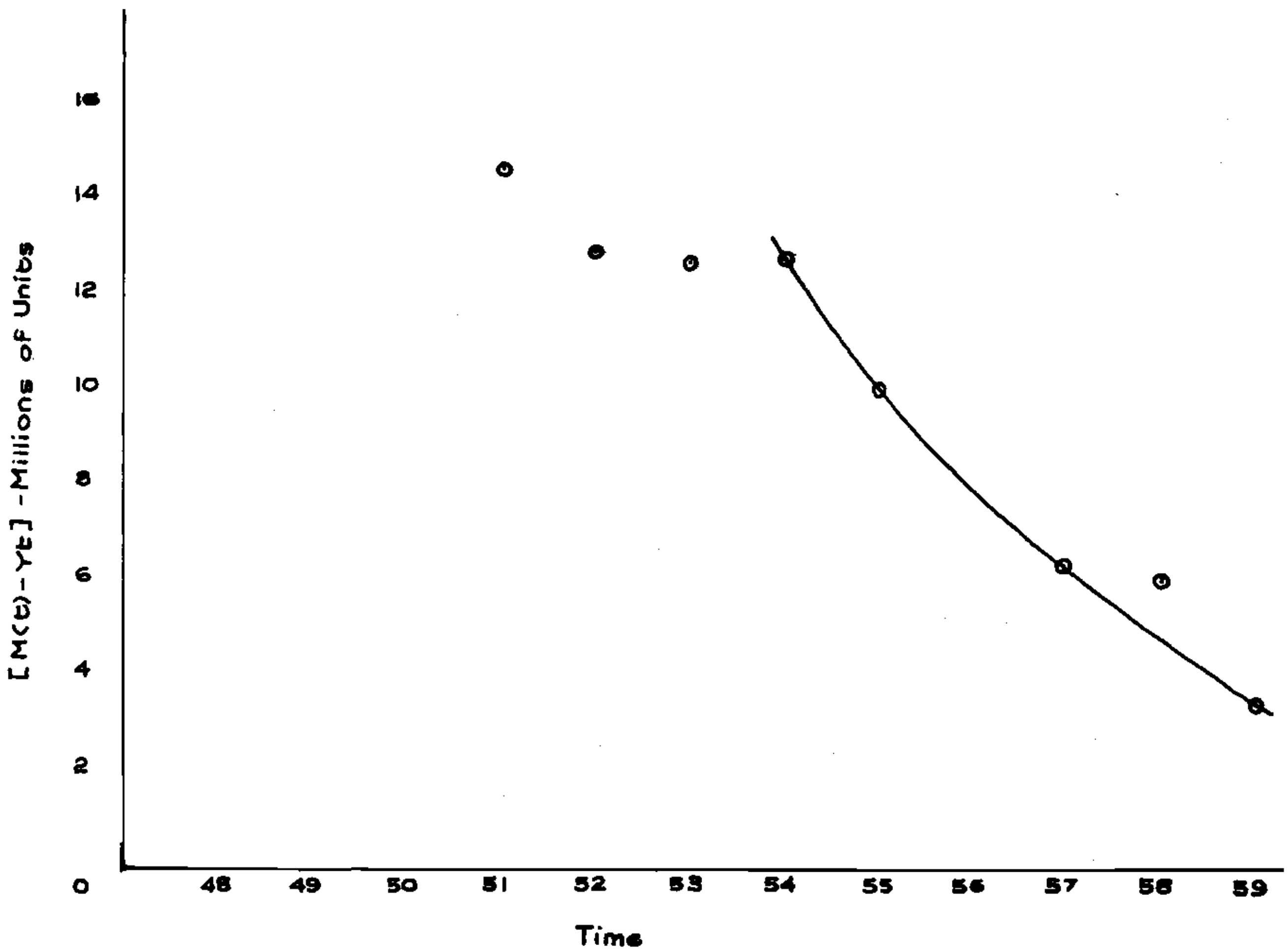


Figure 7. Potential Demand vs. Time

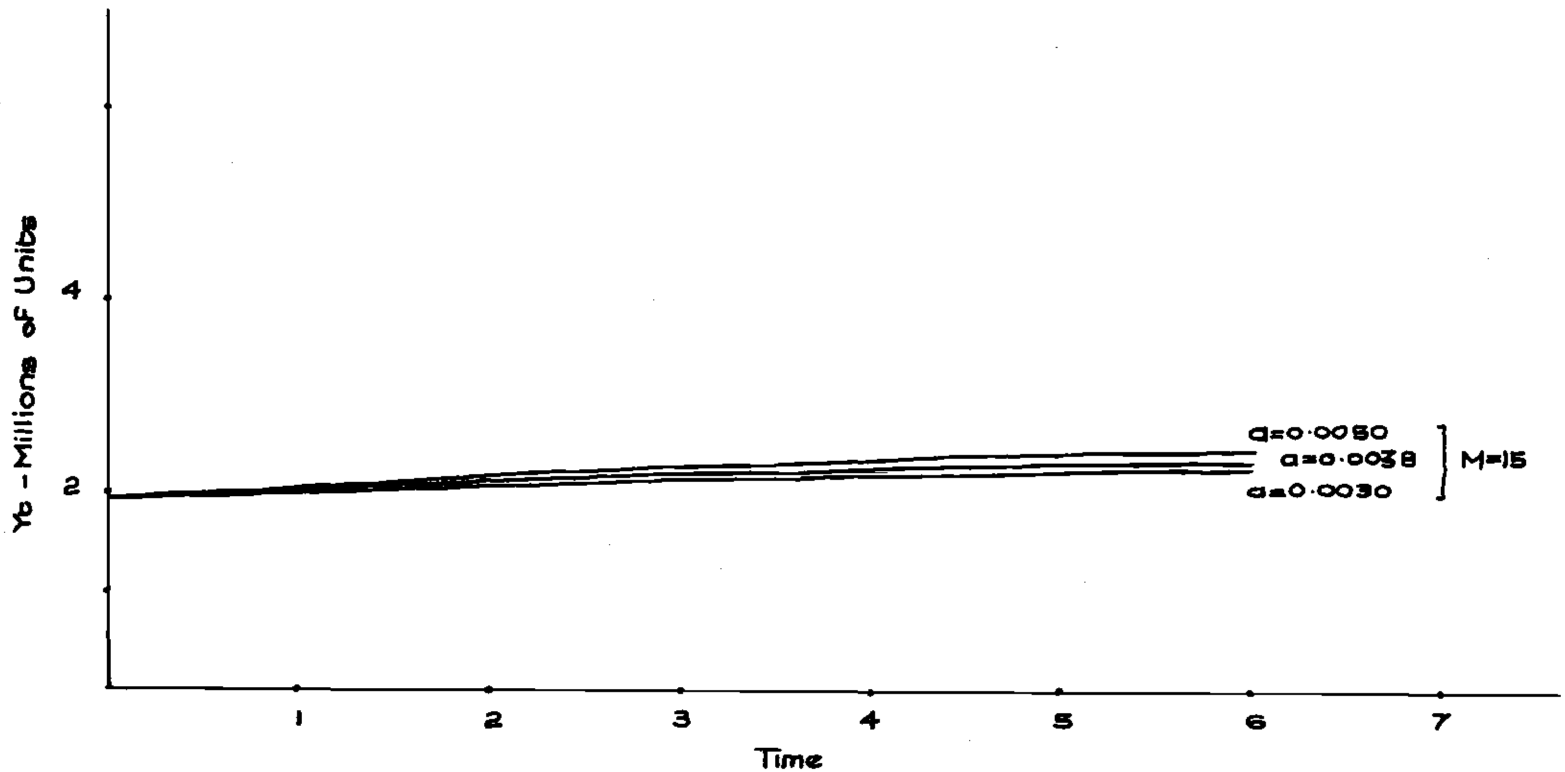


Figure 8 Sensitivity Analysis

(a change of -22 per cent to +34 per cent, about the nominal value of 0.0038), the corresponding changes in Y range only from -4 per cent to +6 per cent. From the above we can conclude that Y is insensitive to changes in "a" and any small errors in estimating this parameter do not invalidate the model.

Effect of Changes in M

It has been seen that small errors in the estimation of "a" do not invalidate the model. Now the changes in maximum ownership level M will be studied. While determining the parameter "a" we have assumed that M can be taken as a constant for the period under study. To test the effect of changes in M, the program shown in Figure 6 is used. The value of M is changed from 12 million to 15 million, which is the range obtained from calculations shown in Table 5. Values of 0.0030, 0.0038, and 0.0050 are chosen for the parameter "a". The recorded computer solutions are shown in Figures 9 and 10. It is found that in the worst case (where a = 0.0050) the change of 20 per cent in maximum ownership level, M, produces a change of only 6 per cent in Y. Thus the assumption of M being constant for a period of six years does not invalidate our basic model.

The results obtained from model shown in equation (15) are plotted in Figure 11. It can be seen that the predicted demand lags behind the actual sales, but the

Table 5. Potential Demand Data

Year	From Table 4		In Millions of Units	
t	a M(t)	a $\left[M(t)-Y_t\right]$	M(t)	$\left[M(t)-Y_t\right]$
1948	46.4	46.12	12.20	12.13
1949	48.9	48.34	12.85	12.70
1950	45.8	44.85	12.05	11.81
1951	57.1	55.43	15.03	14.60
1952	51.9	49.34	13.63	12.96
1953	52.6	48.62	13.82	12.78
1954	56.7	48.85	14.90	12.84
1955	51.5	38.65	13.52	10.160
1956	48.0	30.52	12.63	8.06
1957	48.4	24.40	12.72	6.45
1958	51.5	21.18	13.54	6.12
1959	47.0	12.54	12.38	3.38

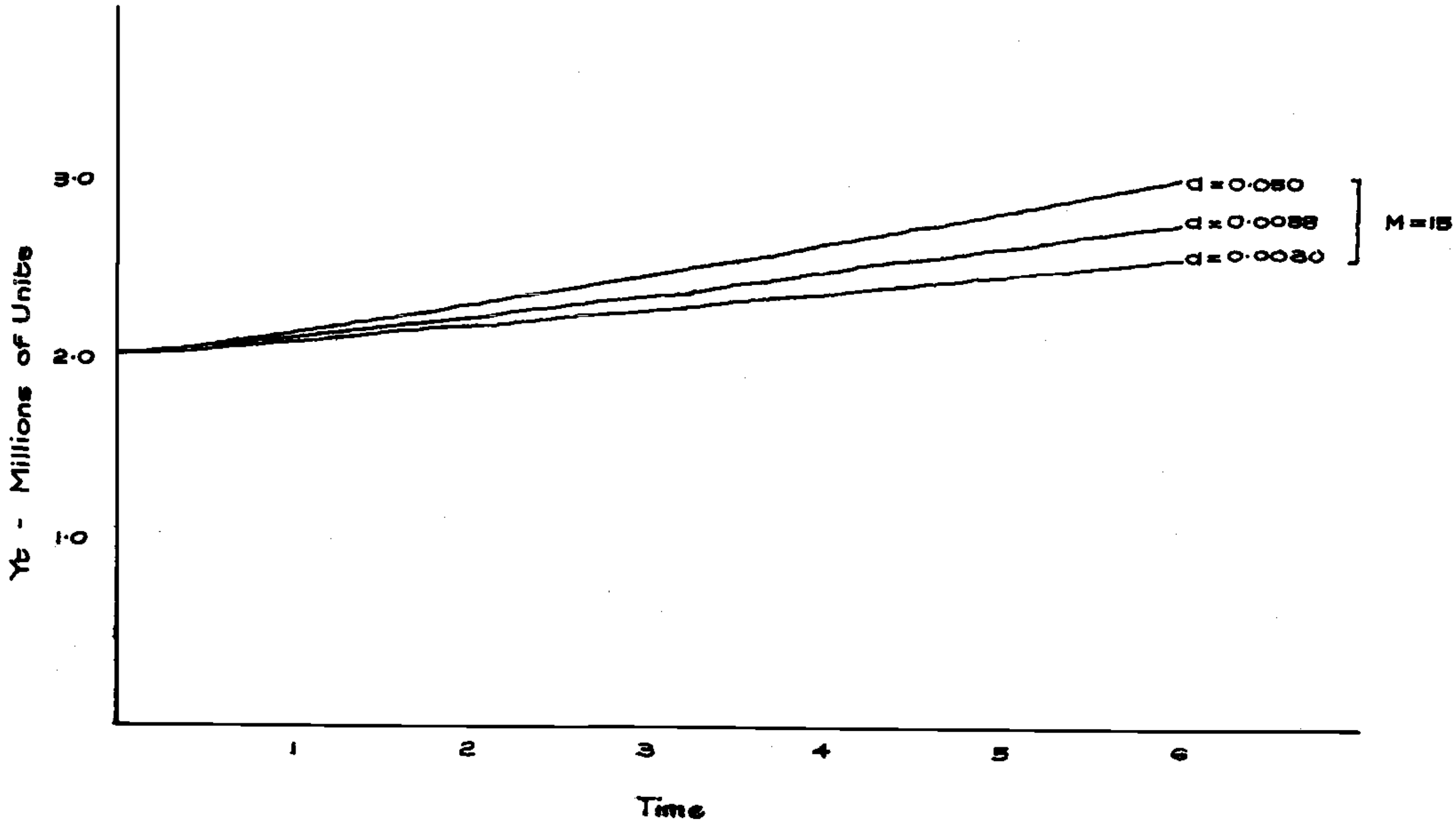


Figure 9 Sensitivity Analysis

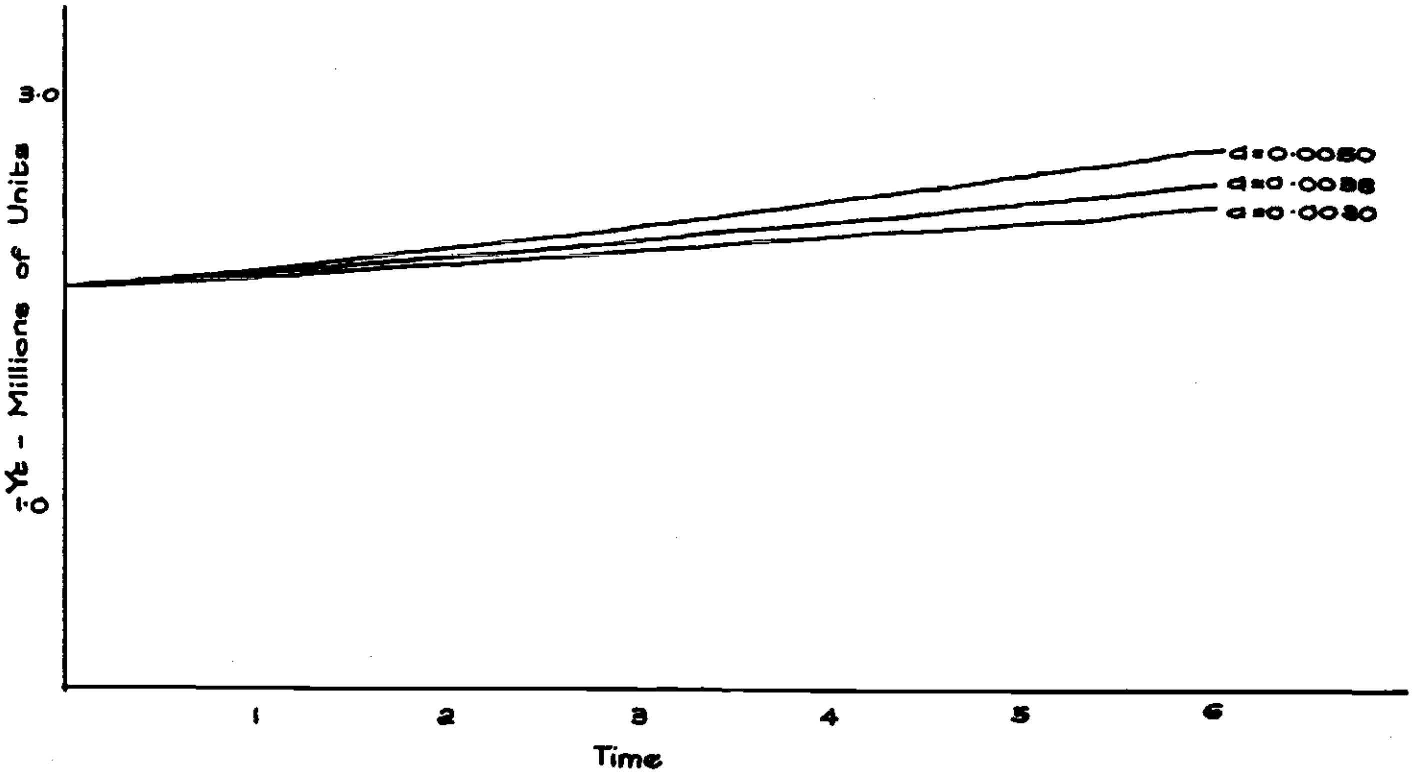


Figure 10 Sensitivity Analysis

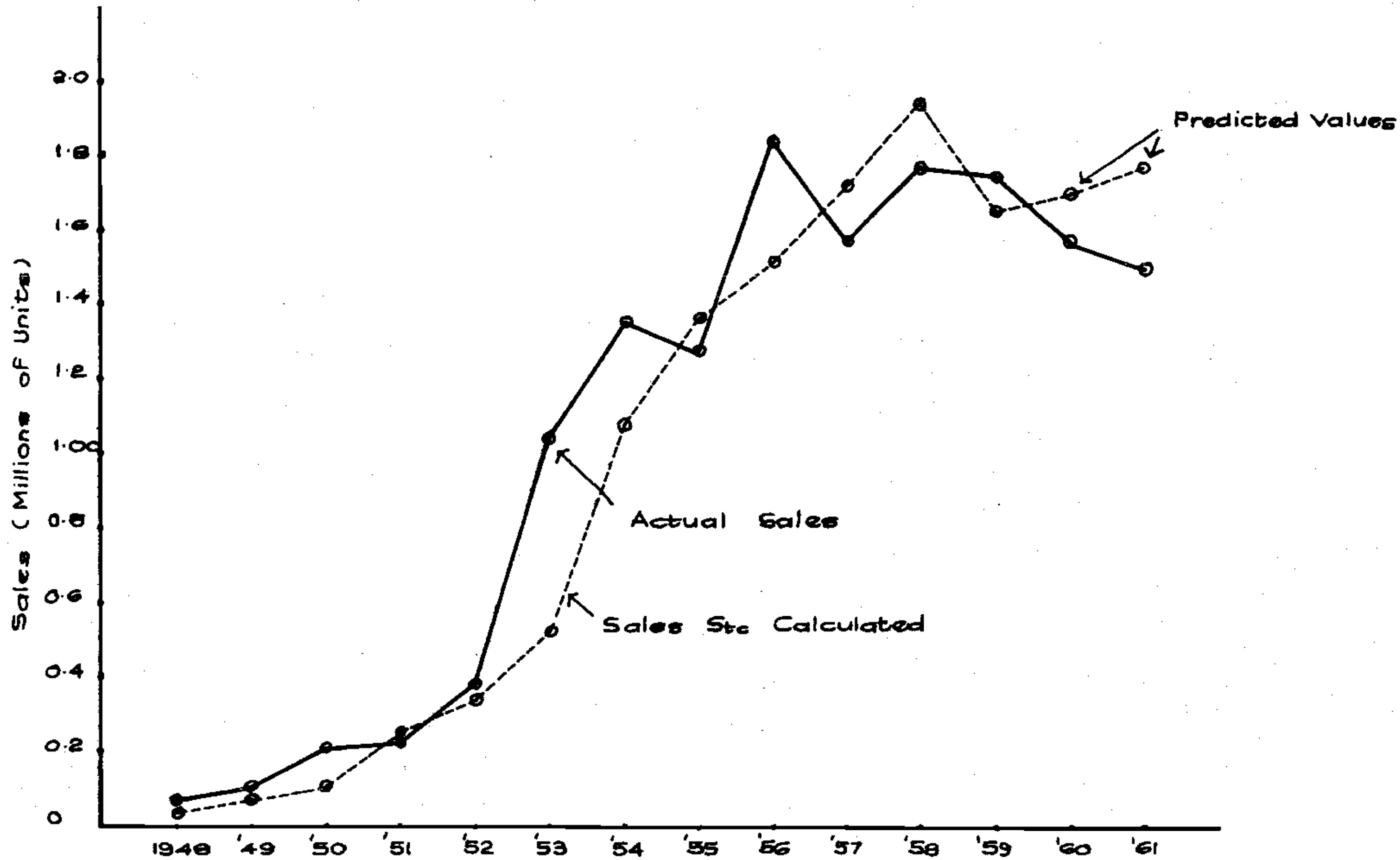


Figure 11. Comparison of Actual and Predicted Sales.

agreement between the actual data and calculated values is much better than in the case of regression models developed in Chapter II.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

In this study several econometric models were developed to predict the demand for room air conditioner units in the United States. Three linear regression models were developed with sales as the dependent variable and a combination of the independent variables--(1) disposable personal income, (2) the change in personal income from previous years, (3) the number of wired dwelling units, and (4) time. The three regression models developed:

1. have a high multiple correlation coefficient of 0.92 and over,
2. have a high standard error which makes the models not as reliable as expected, and
3. do not predict the turning points.

The regression models have not been developed on the basis of a sound economic theory. Consequently, a sound demand theory was developed for the electrical appliances. The model developed for the demand for room air conditioner units on the basis of the developed theory is:

$$St = \left[Y_t - y_{t+1}\right] + \left[\left\{4.57 - \frac{3.733\left(\frac{I_t + 3C_t}{P_t}\right)}{10^{0.01818t-33.1143}}\right\} H_t - 0.0038Y_t\right] Y_t$$

The above model has given satisfactory predictions for the years 1960 and 1961. It has predicted the violent fluctuations in sales better than the linear regression models.

To test the effects of changes in the values of parameter "a" and maximum ownership level M of equation (2) a sensitivity analysis has been carried out with the aid of analog computer. Small errors in the measurement of "a" and M have been found not to affect the model.

In the course of this study, only a small number of observations were available, making the models developed valid only for the near future years. After a few years, when sufficient observations are available, it is recommended that the models be reconstructed so that they can be useful for long-range forecasts also.

The models developed are useful only to predict annual demand. They are not useful either to predict semi-annual or quarterly demand. It would be helpful for production planning purposes to collect data quarterly and develop models to predict demand on a quarterly basis.

In the present models, the effect of weather has not been considered explicitly. It is not possible to develop a single measure for the whole country to represent weather conditions as they vary considerably from region to region at any particular time. But discomfort indexes can be obtained for different regions based on wet and dry bulb temperatures. The effect of weather can be studied for each region and predictions of demand can be made for each region. Their sum will be the demand for the whole United States.

The effect of promotional expenditures like advertising campaigns were not studied and hence are not included in the model. It is recommended that promotional campaigns be included in the future model by taking the total expenditures on these activities as a measure. Alternatively, any other suitable measure can be taken to represent the effect of promotional activities. Then the model will be more realistic and hence more useful.

APPENDICES

Appendix 1

Survival Table for Air Conditioners

Year t	St	1947	1948	1949	1950	1951	1952	1953	1954	1955
	All in Thousands of Units									
1946	30	30	30	30	29.7	28.7	26.2	21.5	15.0	8.5
1947	43		43	43	43.0	42.5	41.2	37.6	30.8	21.5
1948	74			74	74.0	74.0	73.2	70.8	64.7	47.3
1949	89				89.0	89.0	89.0	88.0	85.2	77.8
1950	201					201.0	201.0	200.6	198.8	192.4
1951	238						238.0	238.0	237.5	235.4
1952	380							380.0	380.0	379.2
1953	1,045								1,045.0	1,045.0
1954	1,353									1,353.0
1955	1,275									
1956	1,828									
1957	1,586									
1958	1,673									
1959	1,660									
1960	1,580									
1961	1,500									
y_t		0	30	73	146.7	234.2	430.6	656.5	1,012.0	2,007.1
Y_t		30	73	147	235.7	435.2	668.6	1,036.5	3,360.1	4,566.9
$Y_t - y_{t+1} = R_t$			0	0.3	1.5	4.6	12.1	24.5	49.9	68.2

Appendix 1 (Continued)

Survival Table for Air Conditioners

Year t	1956	1957	1958	1959	1960	1961
	All in Thousands of Units					
1946	3.8	1.3	0.3	0	0	
1947	12.2	5.4	1.8	0.5	0	
1948	37.0	21.0	9.3	3.2	0.8	0
1949	63.7	44.5	25.3	11.2	3.8	1.0
1950	175.7	143.9	100.5	57.1	25.3	8.6
1951	227.8	208.0	170.4	119.0	67.6	30.0
1952	375.8	363.7	332.1	272.1	190.0	107.9
1953	1,042.9	1,033.5	1,000.0	913.3	748.2	522.5
1954	1,353.0	1,350.3	1,338.1	1,294.8	1,182.5	968.7
1955	1,275.0	1,275.0	1,272.5	1,261.0	1,220.2	1,114.4
1956		1,828.0	1,828.0	1,824.3	1,807.9	1,749.4
1957			1,586.0	1,586.0	1,582.8	1,548.6
1958				1,673.0	1,673.0	1,669.7
1959					1,660.0	1,660.0
1960						1,580.0
1961						
y_t	3,291.9	4,446.6	6,078.3	7,342.5	8,502.1	9,400.8
Y_t	4,566.9	6,274.6	7,664.3	9,015.5	10,162.1	10,980.8
$Y_t - y_{t+1} = R_t$	120.3	196.3	321.8	513.4	761.3	1,027.5

Appendix 2

Deviations of Real Purchasing Power

Year t	In Billions of Dollars Disposable Income I_t	Net Credit Extended (Excluding Autos) C_t	Price Index of House Furnishings 1947-49=100 P_t	Real Purchasing Power (in Billions) $\left[I_t+3C_t\right] P_t$	Real Purchasing Power Trend	Deviations from Trend $\frac{\frac{I_t + 3C_t}{P_t}}{f(t)}$
1930	74.4	-0.07	64.9	114.3		
1931	63.8	-0.18	57.6	109.8		
1932	48.7	-0.36	50.6	94.1		
1933	45.7	-0.07	48.6	93.6		
1934	52.0	0.16	50.9	103.1		
1935	58.3	0.45	50.0	119.3		
1936	66.2	0.54	50.8	133.5		
1937	71.0	0.27	55.0	130.6		
1938	65.7	0.07	54.5	120.9		
1939	70.4	0.41	53.4	136.0		
1940	76.1	0.44	53.0	147.9		
1941	93.0	0.19	56.6	165.3		
	Excluding War Years					
1946	159.2	1.18	83.9	193.9	183.6	1.056
1947	169.0	1.58	97.2	178.7	191.5	0.933
1948	187.6	1.21	103.2	185.3	200.7	0.923
1949	188.2	1.06	99.6	192.1	208.2	0.922
1950	206.1	1.59	100.3	210.2	217.2	0.968
1951	226.1	0.69	111.2	205.2	226.4	0.906
1952	237.4	2.35	108.5	225.3	236.1	0.954

Appendix 2 (Continued)

Deviations of Real Purchasing Power

Year t	In Billions of Dollars Disposable Income I_t	Net Credit Extended (Excluding Autos) C_t	Price Index of House Furnishings 1947-49=100 P_t	Real Purchasing Power (in Billions) $\left[I_t+3C_t\right] P_t$	Real Purchasing Power Trend	Deviations from Trend $\frac{\frac{I_t + 3C_t}{P_t}}{f(t)}$
1953	250.2	1.50	107.9	236.5	246.2	0.961
1954	254.5	0.59	106.1	241.3	256.7	0.941
1955	270.2	1.73	104.1	264.5	267.7	0.988
1956	287.2	1.95	103.0	284.5	279.1	1.019
1957	307.9	1.34	104.6	298.2	291.1	1.024
1958	316.5	1.05	103.9	307.7	303.5	1.014
1959	334.6	3.48	103.9	330.8	317.5	1.042
1960	349.4	2.40	104.2	342.3	328.8	1.038
1961	363.6	1.2	103.6	358.6	344.1	1.042

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